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DYNAMIC ELASTO-PLASTIC ANALYSIS OF A SIMPLE SYSTEM.(U)
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Dynamic Elasto-Plastic Analysis of a Simple System

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Structures Group
Ocean Technology Division

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Six design categories were applied to design a three-bar structure connecting two rigid masses. The structure was taken as redundant, made of an elasto-plastic material, and required to resist dynamic loads applied independently from two directions. Weights and deformations of the structures are compared. Two of the categories allow significant reduction in the weight of the structure without producing excessive deformation.

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DYNAMIC ELASTO-PLASTIC ANALYSIS OF A SIMPLE SYSTEM

INTRODUCTION

More than ten years ago, the Dynamic Design Analysis Method[1] (DDAM) was developed at the Naval Research Laboratory (NRL) as a tool for the designers of surface ship and submarine heavy weight equipment systems. This is essentially a linear elastic technique which utilizes normal mode theory and a set of emperically derived shock design values to predict a set of numbers which are then compared with failure criteria.

It was recognized at that time that not all equipment foundation systems had to remain elastic during the shock loading, so an effective yield stress, and a reduced design value set was provided for these cases.

Many engineers have felt for some time that the limit design approach would prove beneficial in limiting acceleration values, and lower foundation weights. However the problems associated with structural interactions in plastic dynamic design, particularily with respect to shock design values for multi-degree of freedom systems have so far been intractable.

It is the purpose of this report to analyze a simple system under DDAM like conditions and to see the effects of yielding in this

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hypothetical case. The input structural interaction problem can be solved for a two-mass single spring system subjected to impulse, and the effects of various design rules, and arbitrary reduction of foundation weight presented in formula and graphical form.

Two other techniques are also presented and compared. They are a constant foundation weight design and a predetermined deflection design.

THE PROBLEM

Consider the system of Fig. (1). The problem addressed in this report is to design the foundation system to have minimum weight, and then to examine the consequences of this design on deflections, and accelerations, and to compare with other design values.

It is assumed that the foundation is composed of prismatic bars, aligned in such a fashion that their lines of action pass through the center of gravity of the equipment, and that the system is restrained from rotation during lateral shock. No bending of the bars occurs. The central bar has length L.

The possibility of elastic, elasto-plastic or plastic buckling is not considered.

The material considered is ductile, with a pronounced range of yielding without strain hardening.

THE DESIGN PROBLEM.

The inputs have been chosen to be impulsive so that the interaction problem becomes solvable. The foundation is to be designed

by a simple set design rules for each category of importance and performance requirement.

In an effort to create an analog for the Dynamic Design Analysis Method's (DDAM) allowable yield stress, it will be assumed that for vertical motion the foundation acts as a single section rather than three separate ones. This would not be allowed under current rules because they are pure tension-compression members, but this will be overlooked in this hypothetical case to study the effects of increased allowable yield.

In addition two other techniques will be analyzed. The first of these is to choose the minimum weight required to resist the Horizontal or Vertical Shock, and the second is to choose a design such that gross yielding impends in both horizontal and vertical directions.

DESIGN CATEGORIES

Category I

The system shall remain elastic under the action of full shock inputs.

Category II

Fullshock inputs shall be used and the allowable yield stress shall be based upon the formula

$$\sigma_a = \sigma_y + F\,[\sigma_u - \sigma_y] \,, \qquad (1)$$

where F is the ratio of the fully plastic load to the load at which yielding first occurs, minus one. That is to say

$$F = P_p/P_e - 1. \quad (2)$$

Category III

The shock design value shall be divided by 2, and under this condition no yielding is permitted (no allowable increased yield).

There are two possible cases here:

1. Both Horizontal and Vertical Shock Design Values are divided by two, or
2. Only the Vertical Shock Design Value is divided by two.

Category IV

The shock design value shall be divided by 2 and increased yield is allowed.

There are two possible cases here:

1. Both Horizontal and Vertical Shock Design Values are divided by two, or
2. Only the Vertical Shock Design Value is divided by two.

Category V

The minimum foundation weight required to resist the greatest shock shall be used, and the columns proportioned to resist the other shock.

Category VI

The system shall be designed so that gross yielding impends in both shock directions at the design levels.

THE INTERACTION PROBLEM

It is assumed:

1. that the foundation weight W can be added to fixed equipment weight W_2 to give the equipment-foundation weight W_1, $(W_1 = W_2 + W)$;

2. No rotations occur;

3. The foundation consists of pin ended columns, the outer ones battered at 45^o;

4. Even though these are columns Eq. (1) holds; (this is different than DDAM).

5. The base and equipment are rigid masses;

6. The shock inputs are generated by an impulse.

Returning to Fig. (1), the initial kinetic energy is

$$E_o = \tfrac{1}{2} M_o \frac{I^2}{M_o^2}. \tag{3}$$

The kinetic energy of translation is:

$$E_T = \tfrac{1}{2} \frac{(M_o + M_1) I^2}{(M_o + M_1)^2}. \tag{4}$$

Therefore the energy available for strain is the difference between them. Let $V_o = I/M_o$,

$$E_s = \frac{V_o^2 M_o M_1}{2[M_o + M_1]}. \tag{5}$$

For elastic response the energy stored in the foundation is,

$$E_f = 1/2\ KX^2 = 1/2\ M_1\ \omega^2 X^2 \qquad (6)$$

where K is the foundation stiffness.

Defining X_ω as the spectral design velocity $(\bar{V})$ results in

$$\bar{V} = \frac{V_o}{\sqrt{1 + \frac{M_1}{M_o}}} = \frac{V_o}{\sqrt{1 + \frac{W_1}{W_o}}} \qquad (7)$$

Eq. (7) is then the elastic spectral velocity necessary for design. Note that it is a function of M_1 which contains the mass of the foundation and the equipment mass.

Eq. (7) has been developed because the DDAM like design rules are based on elastic response.

STATIC ANALYSIS

Assumptions

1. "L" is assumed large with respect to any deflection.
2. Compression and tension properties are the same. Bending is ignored.
3. The deflections of the system are small enough so that changes in stiffness do to geometrical changes can be ignored.

Lateral Motions

In this condition the middle vertical does not contribute to the stiffness so it will be ignored. Consider Fig. 2a. The force T in each bar is found to be

$$T = P/\sqrt{2} = \frac{P\sqrt{2}}{2} \ . \qquad (8)$$

As yield impends the new lengths of each bar are $\sqrt{2}\,L \pm \sqrt{2}\,SL/E$. The energy stored in the bars becomes;

$$E_s = \frac{2\sqrt{2}\,S^2 L\,A_2}{2E} = \frac{\sqrt{2}\,S^2 L\,A_2}{E} , \qquad (9)$$

where A_2 is the area of one bar.

The energy of the Load is $P\,\Delta_{H/2}$, where Δ_H is the horizontal deflection, so

$$\Delta_H = \frac{2\sqrt{2}\,S^2 L\,A_2}{PE} .$$

Now since P was assumed to be the load at which plasticity impends,

$$SA_2 = \frac{P\sqrt{2}}{2} ,$$

so

$$\Delta_H = 2\,\frac{SL}{E} . \qquad (10)$$

Fig. 2b then shows the load deflection curve for lateral motions.

VERTICAL MOTION

In some cases (as will be shown) the minimum foundation weight design does not involve the central bar. In some cases (in the absence of a horizontal shock) the design might not involve the outer bars, and in some cases all bars are needed, so each case will be developed.

Inner Bar Only

In this event only a vertical column would be used as shown in Fig. (3a). At yield its maximum force would be SA_1 and its deflection

would be SL/E. Note that this is 1/2 of Δ_H.

Outer Bars Only

From Fig. (3b), $T = \sqrt{2}\,P/2$. At yield the in line deflection of each bar is $\sqrt{2}$ SL/E so the total deflection becomes

$$\Delta_v = 2\,\frac{SL}{E} \qquad (11)$$

It is interesting to note that the numerical value of Δ_v equals the numerical value of Δ_H.

All Bars Present

Fig. 3c is a sketch of the force system for the combination. Suppose P has a value P_1 such that yield impends in the middle bar. Under these conditions the forces in the bars are $T_1 = SA_1$ and $T_2 = SA_2/2$, and the deflection is δ = SL/E. So

$$P_\epsilon = S\left[A_1 + \frac{\sqrt{2}\,A_2}{2}\right]. \qquad (12)$$

If the force P were made larger the inner bar would yield at constant force and the forces in the outer bars would be raised to SA_2. Then P becomes;

$$P_p = S\,[A_1 + \sqrt{2}\,A_2] \qquad (13)$$

and the deflection equals $\Delta_v = 2\delta$.

For convenience let $q = A_2/A_1$ and $Q = \frac{\sqrt{2}}{2}\,q$, so

$$P_\epsilon = S\,[1 + Q]\,A_1 \qquad (14)$$

and

$$P_p = S\,[1 + 2Q]\,A_1\,. \qquad (15)$$

Foundation Weights

From geometrical considerations the foundation weight can be computed to be:

Outer Bars only,

$$W = 2\sqrt{2}\, L\,\gamma\, A_2 \;; \tag{16}$$

Vertical Bar Only,

$$W = L\,\gamma\, A_1 \;; \tag{17}$$

and the Combination

$$W = 2\sqrt{2}\, L\,\gamma\, A_2 + L\,\gamma\, A_1 \tag{18}$$

$$= L\,\gamma\,[1 + 4Q]\, A_1 \;. \tag{18a}$$

Allowable Yield Stress

Inspection of Fig. 2b illustrates that no increase in the allowable yield stress is permitted for lateral motion.

In the vertical direction, from Eq. (1) and (2),

$$\sigma_a = S + (P_2/P_1 - 1)\,(\sigma_u - S) \tag{19}$$

Assume $[\sigma_u - S] = CS$, which then produces,

$$\sigma_a = S\left[1 + \frac{CQ}{1+Q}\right] . \tag{19a}$$

For some materials $\sigma_u \approx 2S$, so in this case

$$\sigma_a = S\,(1 + 2Q)/(1 + Q) \tag{19b}$$

The imaginary design yield deflection then becomes

$$\delta_i = \frac{SL}{E}\left[1 + \frac{CQ}{1+Q}\right] \tag{20a}$$

for the general case, and

$$\delta_i = \frac{SL}{E}\left[\frac{1+2Q}{1+Q}\right], \qquad (20b)$$

for the previously postulated material.

SHOCK DESIGN

There is an advantage in first doing a simple problem to find the effects of combining various geometries with vertical and lateral shock.

Vertical Shock, Elastic Design

Assume that only the middle bar is present.

Then equating energies yields;

Since

$$E_s = \frac{S^2LA_1}{2E} = \frac{V_v^2 \; M_o}{2\left[1 + \frac{W_o}{W_1}\right]},$$

and since

$$A_1 = \frac{W}{\gamma L},$$

therefore

$$W = \frac{V_v^2 \; M_o \; E \; \gamma}{S^2\left[1 + \frac{W_o}{W_1}\right]}, \qquad (21)$$

is the amount of material necessary for the vertical bar. In these equations

$V_v = I_v/M_o$.

Assume that the middle bar is not present and only the outer bars

exist.

Again

$$ES = \frac{S^2 \sqrt{2}\, L A_2}{E} = \frac{V_v^2 M_o}{2\left[1 + \frac{W_o}{W_1}\right]} ,$$

and since

$$S_2 = \frac{\sqrt{2}\, W}{4 L \gamma} ,$$

therefore

$$W = \frac{V_v^2 M_o E \gamma}{S^2 \left[1 + \frac{W_o}{W_1}\right]} . \tag{22}$$

This is Eq. (21) all over again. This seems to say that from a strength point of view it is immaterial whether or not the material is concentrated in the vertical bar or in the outer bars. Of course the deflection has been doubled, but some resistance to lateral motion has been supplied.

Lateral Shock Elastic Design

Assume a lateral shock of intensity $V_H = I_H/M_o$. Again equating energies and using the weight area relationship results in

$$W = \frac{V_H^2 M_o E \gamma}{S^2 \left[1 + \frac{W_o}{W_1}\right]} \tag{23}$$

This interesting result leads to the conclusion that the middle bar is never needed in elastic design if the bars are proportioned to withstand the greatest shock. In this case the deflection for V_{max} is 2 SL/E, while that of the lesser shock is smaller. In fact the elastic deflection ratio is proportional to the ratio of the V's.

For some classes of elastic design it may be desirable to limit the deflection in the vertical direction. Then a middle bar can be used. Of course there may be an associated weight penality.

DESIGN BY CATEGORY RULES

The procedure here is to define areas of the two outer bars and then the inner bar. In this fashion a q required may be obtained and the foundation weight calculated. Then the features of the design can be examined.

In all cases the weight required to withstand the horizontal shock is

$$W_H = \frac{V_H^2 M_o E \gamma}{S^2 [1 + \frac{W_o}{W_1}]}, \qquad (24)$$

where so far W_1 is unknown. Dividing this weight by $2\sqrt{2}\gamma L$, which is the weight of the outer bars for unit area yields

$$A_2 = \frac{V_H^2 M_o E}{2\sqrt{2} S^2 L [1 + \frac{W_o}{W_1}]} \qquad (25)$$

the area required.

Category I

Elastic design requires that the strain energy be equal to $P_e \delta/2$. Then

$$\left[A_1 + \frac{\sqrt{2}}{2} A_2\right] \frac{S^2 L}{2E} = \frac{V_v^2 M_o}{2 [1 + \frac{W_o}{W_1}]}$$

Solving for A_1 yields

$$A_1 = \frac{V_v^2 M_o E}{S^2 L [1 + \frac{W_o}{W_1}]} - \frac{\sqrt{2}}{2} A_2 \quad .$$

To find the required area ratio

$$\frac{A_2}{A_1} = \frac{V_H^2}{2\sqrt{2}\left[V_v^2 - \frac{\sqrt{2}}{2}\frac{V_H^2}{2\sqrt{2}}\right]}$$

or

$$q = \frac{\sqrt{2}\,({}^{V}H/V_v)^2}{4 - ({}^{V}H/V_v)^2} \quad . \tag{26}$$

Examination of this equation reveals that the middle bar is no longer required to limit the deflection if ${}^{V}H/V_v \geq 2$.

The expression for the required foundation weight can now be found. It is

$$W = \frac{V_v^2 M_o E \gamma}{S^2 [1 + \frac{W_o}{W_1}]} \left[1 + \frac{3}{4}\left(\frac{V_H}{V_v}\right)^2\right] . \tag{27}$$

This shows a weight penalty when compared with Eq. (21) to reduce the deflection to δ in the vertical direction.

Category II

The same procedure can be followed and it results in

$$q = {}^{A}2/A_1 = \frac{\sqrt{2}}{2} \; \frac{1 - \sqrt{1 - \left(\frac{V_H}{V_v}\right)^2}}{\sqrt{1 - \left(\frac{V_H}{V_v}\right)^2}} \quad . \tag{28}$$

It is of interest to note here that Category II design can only be used if ${}^{V}H/V_v < 1$.

The expression for the required foundation weight can now be found, and it is

$$W = \frac{V_v^2 M_o E \gamma}{S^2 \left[1 + \frac{W_o}{W_1}\right]} \left[\frac{1 + \left(\frac{V_H}{V_v}\right)^2 + \sqrt{1 - \left(\frac{V_H}{V_v}\right)^2}}{2} \right]. \tag{29}$$

The function of V_H/V_v in the brackets is positive and greater than one for $0 \leq \left(\frac{V_H}{V_v}\right) \leq 1$. Its greatest value occurs at

$$V_H/V_v = \sqrt{2} / 2 \quad \text{and is } [3 + \sqrt{2}] / 4 \approx 1.104.$$

Design by Category II is only possible if $V_H/V_v < 1$ and a weight penalty is paid, even though the design deflection exceeds δ and some permanent set will occur.

Category III

In this category there are two cases.

Case 1.

Both V_H and V_v are divided by two. This results in

$$W = \frac{V_v^2 M_o E \gamma}{S^2 \left[1 + \frac{W_o}{W_1}\right]} \; \frac{\left[1 + \frac{3}{4}\left(\frac{V_H}{V_v}\right)^2\right]}{4}, \tag{30}$$

Where V_v^2 has not been changed, but $(1/2)^2$ inserted.

There will be some permanent set and a small weight penalty when compared to the two bar case.

Case 2.

This case occurs if the horizontal shock design value is not

reduced. Then

$$q = \frac{\sqrt{2}\left(\frac{V_H}{V_v}\right)^2}{1 - \left(\frac{V_H}{V_v}\right)^2}, \tag{31}$$

and

$$W = \frac{V_v^2 M_o E \gamma}{S^2 \left[1 + \frac{W_o}{W_1}\right]} \left[\frac{1 + 3\left(\frac{V_H}{V_v}\right)^2}{4}\right] \tag{32}$$

where V_v and V_H are the <u>original</u> values, not divided by two.

Category IV

Case 1 - Both V's divided by 2, V_v = original value.

$$W = \frac{V_v^2 M_o E \gamma}{S^2 \left[1 + \frac{W_o}{W_1}\right]} \left[\frac{1 + \left(\frac{V_H}{V_v}\right)^2 + \sqrt{1 - \left(\frac{V_H}{V_v}\right)^2}}{8}\right] \tag{33}$$

Case 2 - Only V_v divided by 2,

If the previous procedure is used then

$$q = \frac{\sqrt{2}}{2} \frac{\left(1 - \sqrt{1-4\left(\frac{V_H}{V_v}\right)^2}\right)}{\sqrt{1 - 4\left(\frac{V_H}{V_v}\right)^2}}, \tag{34}$$

V_v = original value.

Note that design is only possible if the original $V_H/V_v < 1/2$.

The foundation weight can now be found to be:

$$W = \frac{V_v^2 M_o E \gamma}{S^2 \left[1 + \frac{W_o}{W_1}\right]} \frac{\left[1 + 4\left(\frac{V_H}{V_v}\right)^2 + \sqrt{1 + 4\left(\frac{V_H}{V_v}\right)^2}\right]}{8}, \tag{35}$$

where again V_v and V_H are the full scale values. Again a weight penalty is being paid with respect to the two bar case.

For those designs in this category where $V_H/V_v > 1/2$, then only

two bars are needed to satisfy the criteria.

Category V

In the two outer bar only case a weight was found that was required to withstand the shock, as shown in Eq. (22). Suppose that for those cases in which $V_H < V_v$ the outer bars were proportioned to resist the horizontal shock and the remaining weight (corresponding to this two bar case) were used to supply the material for the middle bar. The vertical deflection under the shock would exceed δ, but not 2δ, for the full shock input. In contrast then to categories I and II of the Design Rules there would be no weight penalty. However unlike Category I some permanent set would occur.

Since it is assumed $V_H < V_v$ first find the required W from Eq. (22). Note that $W_1 = W_2 + W$ so either a quadratic must be solved, or a numerical interaction technique used.

The weight required to resist the horizontal shock is that shown in Eq. (24), where W_1 has been previously found from Eq. (22). Since $W > W_H$ this additional material can be used in the vertical bar. So

$$W_v = W - W_H.$$

It is now possible to find the required areas. The area ratio is

$$q = \frac{2\left(\frac{V_H}{V_v}\right)^2}{4\left[1 - \left(\frac{V_H}{V_v}\right)^2\right]} \tag{36}$$

It is interesting to note in passing, that for this geometry and these assumptions, that this q is one-half of the q for Category III, Case 2.

Category VI Pre-determined Deflection Design

This approach is to simply increase the design deflection in the vertical direction until it is equal to the horizontal deflection, and gross yielding impends. Of course any value could be selected (the value 1/2 was selected in Category I) but it seems logical to choose this here because at this value ($\Delta = 2\delta$), full plasticity begins. At this condition the energy present in the bars become

$$E_s = \frac{3S^2 L A_1}{2E} + \frac{\sqrt{2}\, S^2 L A_2}{2E} .$$

Then the required area ratio becomes

$$\frac{A_2}{A_1} = \frac{3\sqrt{2}\left(\frac{V_H}{V_v}\right)^2}{4\left[1 - \left(\frac{V_H}{V_v}\right)^2\right]} \tag{37}$$

The foundation weight is found to be

$$W = \frac{V_v^2 M_o E \gamma}{S^2 \left[1 + \frac{W_o}{W}\right]} \left[\frac{1 + 2\left(\frac{V_H}{V_v}\right)^2}{3}\right] . \tag{38}$$

This is less than or equal to the weight of the two bar case, elastic design.

EFFECT OF DESIGN CHOICE

Foundation Weights

In order to have some numerical examples to obtain a feeling for the effects of these design methods, a hypothetical case has been chosen. The foundation material is steel with the following

properties:

γ = 7689 kg/m^3

S = 276 MPa

σ_u = 552 MPa

E = 207 GPa,

and is very ductile with a tensile test load deflection curve that has a large yield deflection (15 or more elastic deflections) without strain hardening.

The base and equipment systems can be considered to be solid rigid masses, the value of the base mass being 1753 kg. The full scale vertical shock impulse is 5343 N.s, giving an initial velocity to the base mass of 3.048 m/s. Fig. (4) is a graph showing the weight $\underline{W}$ of an elastic foundation using only the two outer bars, as given by Eq. (22), for the hypothetical case.

Weights of foundations designed according to the different categories are charted in Fig. (5). The weights were obtained by evaluating Eq. (27), (29), (30), (32), (33), and (38). The area of the center bar becomes zero when V_H/V_v is one-half for category IV-2. The broken line on the chart indicates that the center bar has been removed for larger values of V_H, leaving a two-bar design.

Design Deflections

Maximum vertical deflections of the structures are graphed in Fig. (6) for each category. A deflection ratio of 1.0 would initiate yield in the center bar and a ratio of 2.0 would yield the two outer bars.

Critical Velocities

The vertical impulsive velocity which would initiate yield in the center bar is the first critical velocity. It is given by the relation

$$(V_1/V_v)^2 = (W_f/W) - (3/4)\,(V_H/V_v)^2, \tag{39}$$

where W_f is the weight of a three-bar structure designed by any of the categories for impulsive velocities V_v and V_H, and W is the weight of the elastic two-bar structure from Eq. (22). The relation is graphed for each category in Fig. (7). The first critical velocity is lower than the design velocity V_v unless the structure is heavier than the elastic two-bar foundation.

The second critical velocity brings the two outer bars to yield and the entire structure becomes plastic. The second critical velocity is given by

$$(V_2/V_v)^2 = 3(W_f/W) - 2(V_H/V_v)^2 \tag{40}$$

and is graphed in Fig. (8).

DISCUSSION AND CONCLUSIONS

The objective of the calculations presented here was to gain insight into the relative effectiveness of different goals which can be adopted in designing foundations for equipment. The calculations were made under DDAM-like conditions for purposes of illustration only. They are not applicable to the design of foundation systems for surface ships or submarines. In particular:

1. The two-dimensional foundation of pin-ended bars was chosen so that simple solutions could be obtained in closed form. It does not resemble any practical foundation which could be used on a ship.

2. The material used for the foundation behaves elastically up to its yield stress. This is quite different from the behavior of structural steel, where yield stress is usually defined as the stress which produces 0.2-percent offset in a tensile specimen. Bars of medium steel, for example, would have deformation 2.5 times the elastic deformation as they reached yield stress.

3. The time-history forcing function for shipboard shock can be represented by an equivalent impulse for determining the peak acceleration and deflection of a linear, elastic mode only. The equivalent impulse cannot be used to determine time-history responses or to calculate peak responses of a nonlinear or yielding system, as was done here for the sake of producing a simple illustration.

With the preceding comments kept in mind, a comparison among the design categories can be made. Each category was treated consistent-

ly, if unrealistically, and the comparisons can provide some insight into the relative effectiveness of the rules adopted for each category.

Categories III (reduce inputs) and IV (reduce inputs and allow overstress) produced the lightest-weight foundations, as shown in Fig. (5), but they had the largest deflections under the design input, as indicated in Fig. (6). All four foundations designed to these categories became fully plastic for shock severities less than the level to which they had been designed; see Fig. (8). It can be concluded that neither of these categories achieved the objective of providing lighter-weight foundations with controlled deformation and some safety margin before the onset of a completely-plastic response.

Category II (allow overstress) produced only a moderate weight penalty compared to the elastic two-bar foundation, but its maximum deflections and critical velocities varied greatly with the ratio of the lateral to vertical input. When lateral inputs were small, the foundation designed by Category II approached the robustness of an elastic design (Category I), where an input 73 percent larger than the design value would be required to produce complete plasticity; see Fig. (8). As lateral shock increased, however, the foundation became progressively weaker for vertical shock until it had no overload capability at all when the lateral shock became equal to the vertical.

The weight penalty for Category I (elastic) could become appreciable for lightweight equipment, as indicated by Figs. (4) and (5). An elastic foundation for a 30-kg item, for example, would have a weight ranging from 24 percent to 42 percent of the weight of the equipment.

The heavy foundation is very robust, however, requiring inputs from 1.73 to 1.80 times the design value to produce complete plasticity.

Categories V (design to a predetermined weight) and VI (design to a predetermined deflection) show results which suggest that the other categories may have contained a fundamental defect. The objective of the design exercise was to produce a foundation of lower weight with controlled deflection under shock, but neither weight nor deflection appeared specifically in the rules for Categories I through IV. In each of these categories the design was to a stress, an imaginary stress, a ficticious input, or some combination of these. Designing to parameters other than weight and deflection left both weight and deflection as uncontrolled quantities, which might or might not fall into desired ranges when the design was complete.

Category V provides a foundation which is lighter than either Category I or II, but its deflection, critical velocities, and overload capabilities vary appreciably with the ratio of lateral to vertical design levels.

The foundation of Category VI is lighter in weight than that of Category V. Its deflection and second critical velocity are independent of the lateral design value. It has no design margin, however, since the vertical input used for the design brings it exactly to the onset of complete plasticity. Weight savings compared to the elastic foundation range from 67 percent to 75 percent.

The major insight gained from the calculations is this: If weight and deflection of a foundation are items of special concern, a rational design method should take either or both explicitly into account as bases for the design. Methods based on ad hoc adjustments of other parameters may or may not produce acceptable values of weight and deflection.

REFERENCES

1. R.O. Belsheim and G.J. O'Hara, "Shock Design of Shipboard Equipment I - Dynamic Analysis Method," Naval Research Laboratory NRL Report 5545 (September 16, 1960).

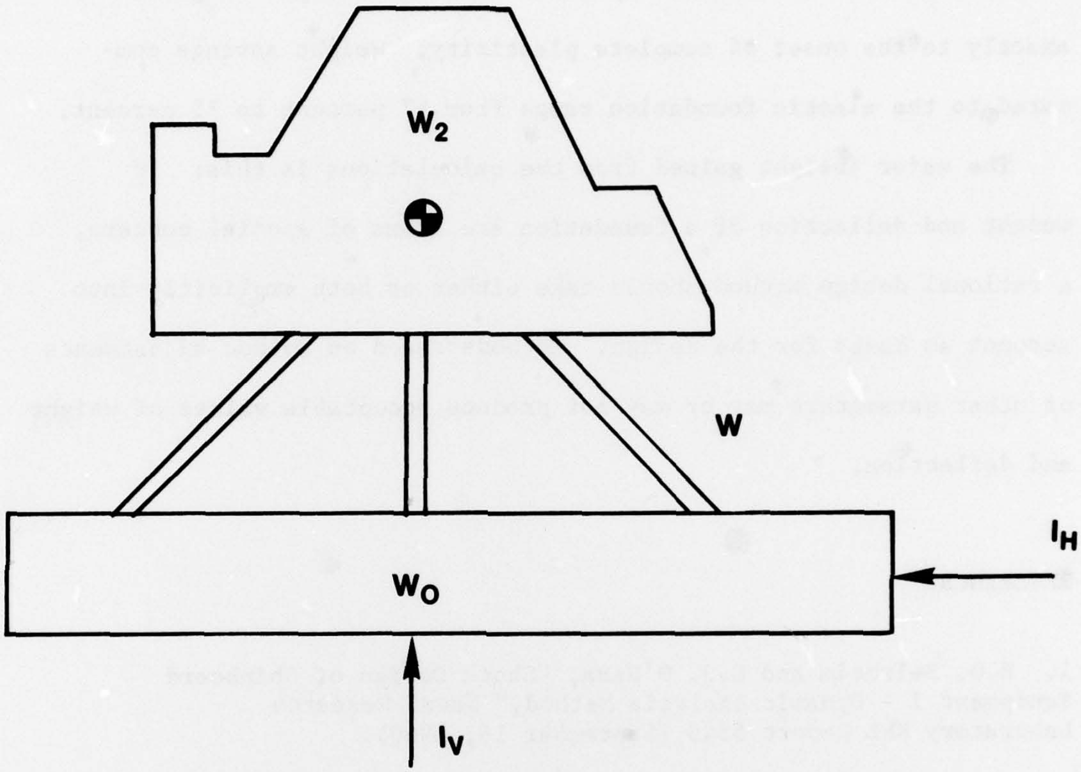

Fig. 1 — The foundation system to be designed. Rigid weights W_0 and W_2 are connected by a three-bar structure of weight W. The base weight is given an impulse which may be directed either vertically or horizontally.

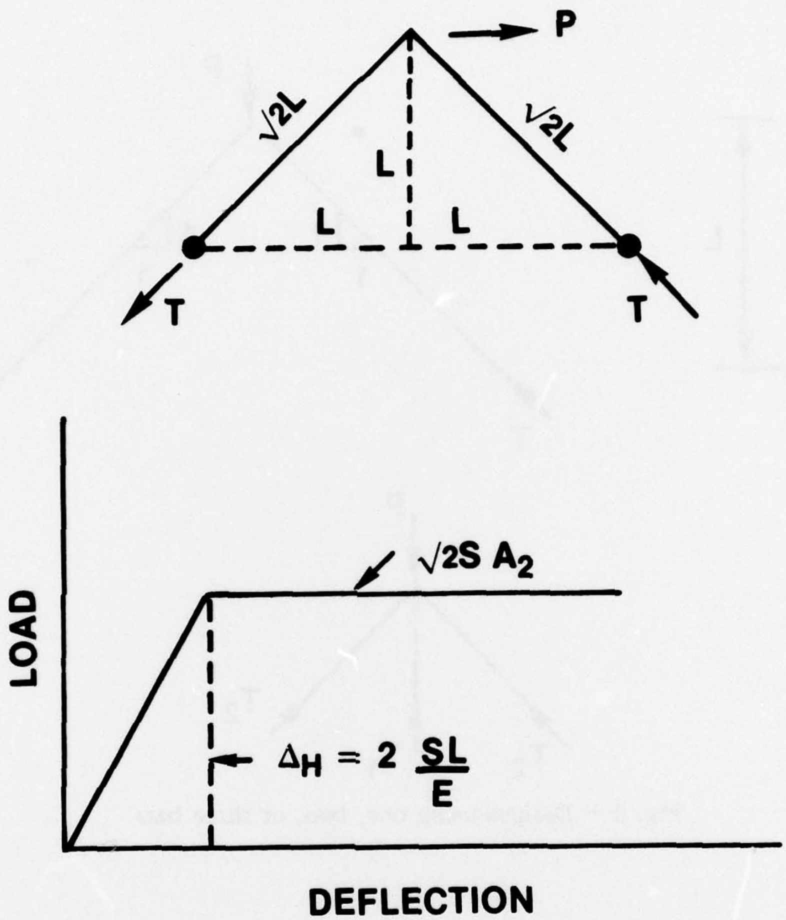

Fig. 2 — Force and deflection curves for lateral motion

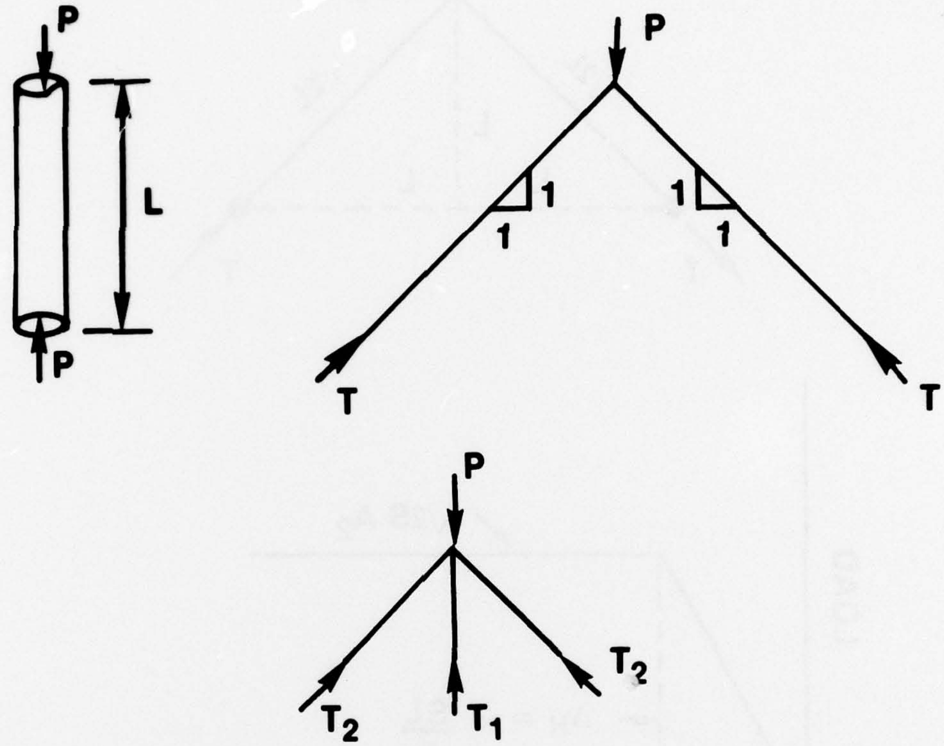

Fig. 3 — Designs using one, two, or three bars

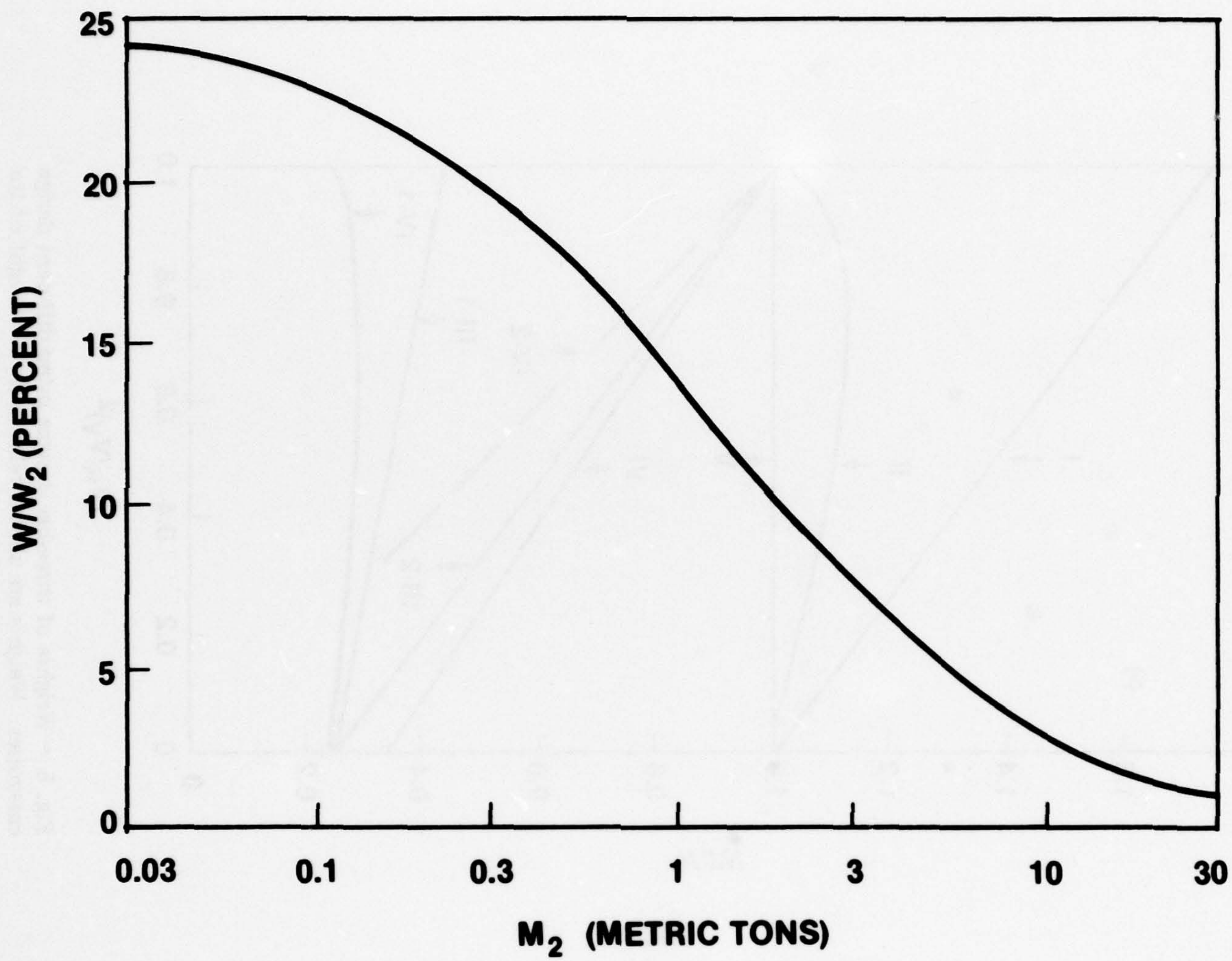

Fig. 4 — Weight of a hypothesized two-bar design

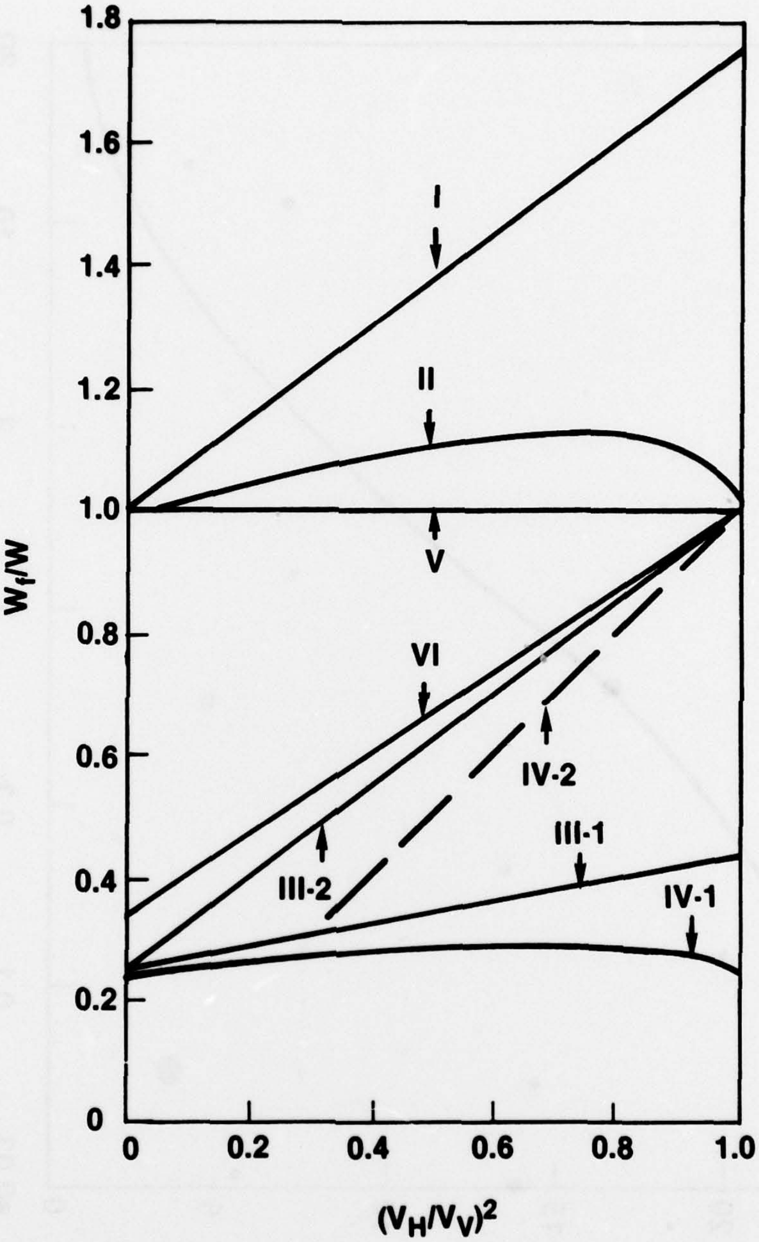

Fig. 5 — Weights of three-bar designs using different design categories. Weights are shown relative to the weight of the two-bar elastic design of the preceding figure.

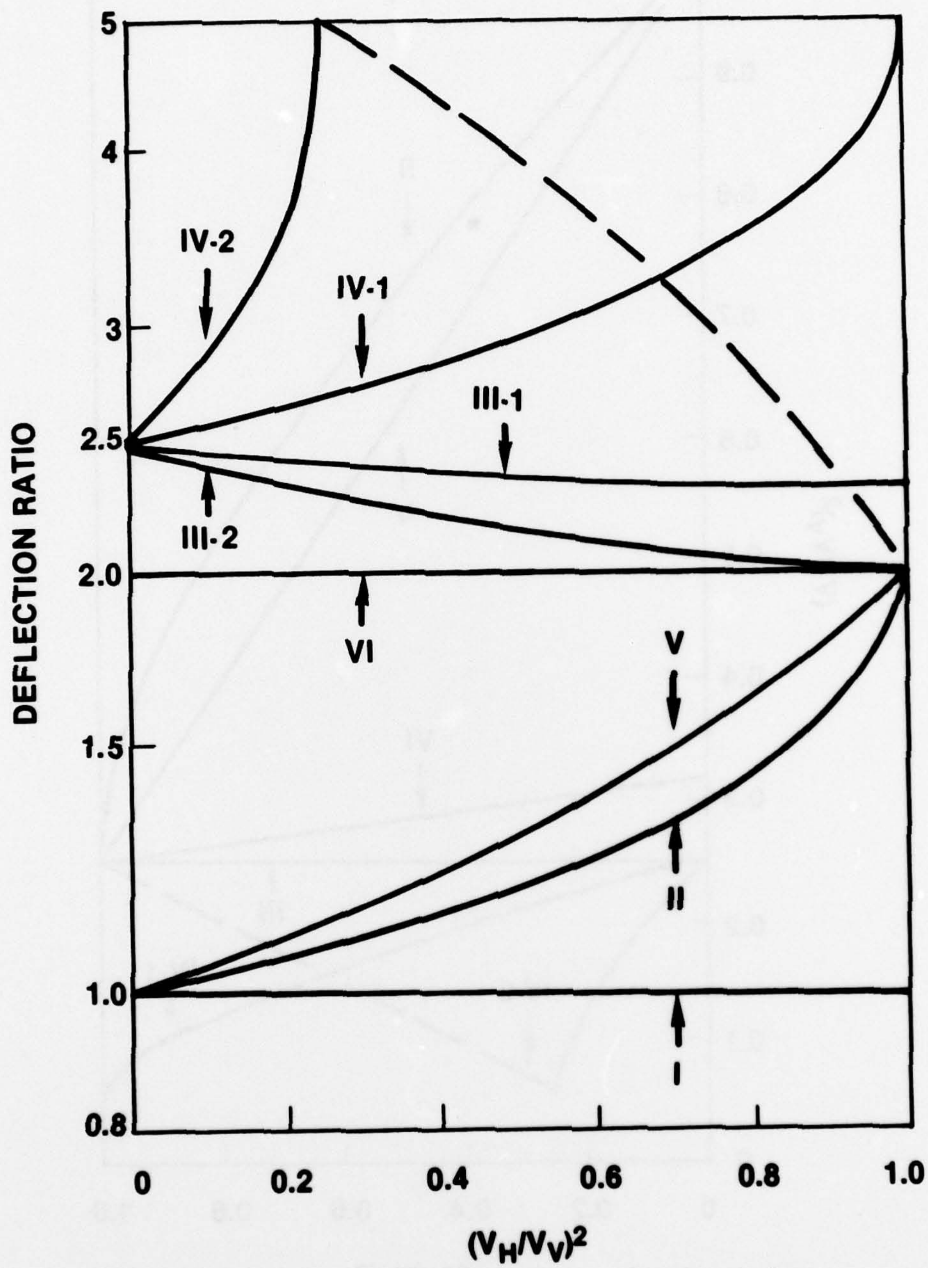

Fig. 6 — Maximum vertical deflections of three-bar designs. Deflections are shown as multiples of the deflection needed to initiate yield in the center bar.

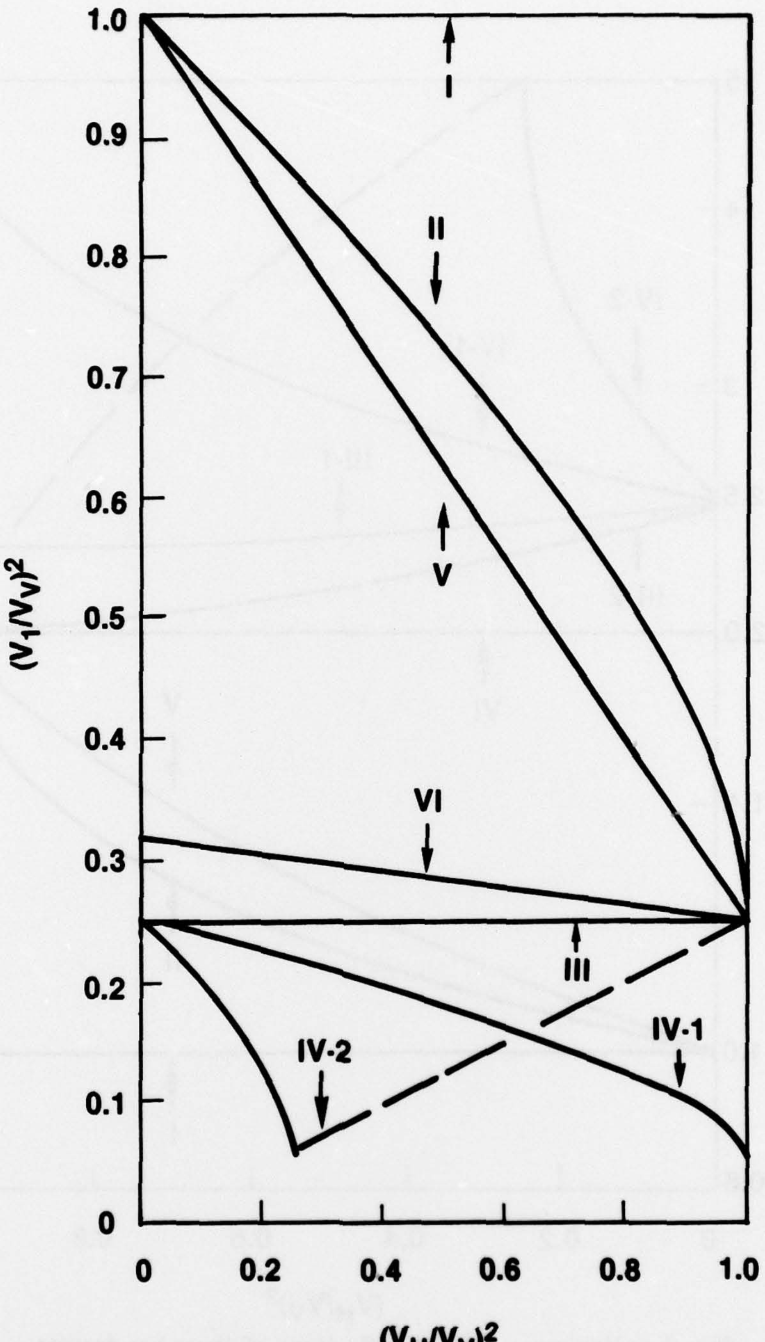

Fig. 7 — First critical velocity. The vertical velocity V_1 initiates yielding in a structure designed to velocities V_v and V_H by categories as labeled.

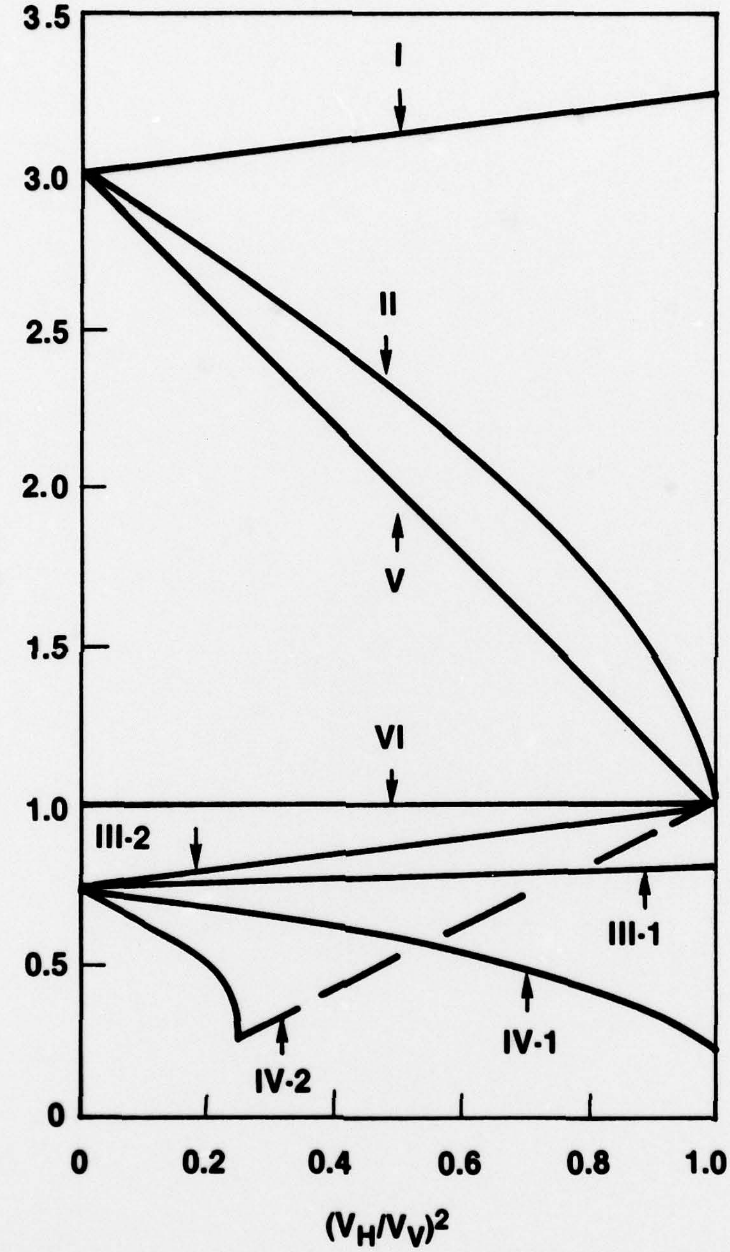

Fig. 8 — Second critical velocity. The vertical velocity V_2 produces completely plastic response of the structure.